

$$\begin{aligned} \textcircled{1} \quad & \int \cos(2x+3) dx \\ & \frac{1}{2} \int 2 \cos(2x+3) dx \\ & \frac{1}{2} \sin(2x+3) + C \end{aligned}$$

$$\begin{aligned} \textcircled{2} \quad & \int \sin(2-3x) dx \\ & -\frac{1}{3} \int \sin(2-3x) (-3) dx \\ & \frac{1}{3} \cos(2-3x) + C \end{aligned}$$

$$\begin{aligned} \textcircled{3} \quad & \int (3x-2)^4 dx \\ & \frac{1}{3} \int (3x-2)^4 (3) dx \\ & \frac{1}{15} (3x-2)^5 + C \end{aligned}$$

$$\begin{aligned} \textcircled{4} \quad & \int x \cos(1-x^2) dx \\ & -\frac{1}{2} \int -2x \cos(1-x^2) dx \\ & -\frac{1}{2} \sin(1-x^2) + C \end{aligned}$$

$$\begin{aligned} \textcircled{5} \quad & \int \frac{2x^3}{1+x^4} dx \quad \begin{array}{l} u = 1+x^4 \\ du = 4x^3 dx \end{array} \\ & \frac{1}{2} \int \frac{4x^3}{1+x^4} dx \\ & \frac{1}{2} \int \frac{1}{u} du \\ & \frac{1}{2} \ln|u| + C \\ & \frac{1}{2} \ln(1+x^4) + C \end{aligned}$$

$$\begin{aligned} \textcircled{6} \quad & \int x(3x+2)^4 dx \quad \begin{array}{l} u = 3x+2 \\ \frac{u-2}{3} = x \\ du = 3 dx \end{array} \\ & \frac{1}{3} \int 3x(3x+2)^4 dx \\ & \frac{1}{3} \int \frac{u-2}{3} u^4 du \end{aligned}$$

$$\begin{aligned} & \frac{1}{9} \int (u^5 - 2u^4) du \\ & \frac{1}{9} \left[\frac{1}{6} u^6 - \frac{2}{5} u^5 \right] + C \\ & \frac{1}{54} (3x+2)^6 - \frac{2}{45} (3x+2)^5 + C \end{aligned}$$

$$\begin{aligned} \textcircled{7} \quad & \int \frac{dx}{1-2x} \\ & -\frac{1}{2} \int \frac{-2 dx}{1-2x} \\ & -\frac{1}{2} \ln|1-2x| + C \end{aligned}$$

$$\begin{aligned} \textcircled{8} \quad & \int \sqrt{3x-2} dx \\ & \frac{1}{3} \int 3(3x-2)^{1/2} dx \\ & \frac{2}{9} (3x-2)^{3/2} + C \end{aligned}$$

$$\begin{aligned} \textcircled{9} \quad & \int x \sqrt{3-2x} dx \quad \begin{array}{l} u = 3-2x \\ du = -2 dx \\ x = \frac{3-u}{2} \end{array} \\ & -\frac{1}{2} \int \frac{3-u}{2} \cdot u^{1/2} du \\ & -\frac{1}{4} \int (3u^{1/2} - u^{3/2}) du \\ & -\frac{1}{4} \left[2u^{3/2} - \frac{2}{5} u^{5/2} \right] + C \\ & -\frac{1}{2} (3-2x)^{3/2} + \frac{1}{10} (3-2x)^{5/2} + C \end{aligned}$$

$$\textcircled{10} \int \frac{\ln x}{x} dx$$

$$\frac{1}{2} (\ln x)^2 + c$$

$$\textcircled{11} \int \frac{\sqrt{1+\frac{1}{x}}}{x^2} dx \quad u = 1 + \frac{1}{x}$$

$$du = -x^{-2} dx$$

$$-\int u^{1/2} du$$

$$-\frac{2}{3} u^{3/2} + c$$

$$-\frac{2}{3} \left(1 + \frac{1}{x}\right)^{3/2} + c$$

$$\textcircled{12} \int x e^{x^2} dx$$

$$\frac{1}{2} \int 2x e^{x^2} dx$$

$$\frac{1}{2} e^{x^2} + c$$

$$\textcircled{13} \int x^3 (x^4 - 1)^2 dx \quad u = x^4 - 1$$

$$du = 4x^3 dx$$

$$\frac{1}{4} \int u^2 du$$

$$\frac{1}{12} (x^4 - 1)^3 + c$$

$$\textcircled{14} \int \frac{x^3}{1+x^2} dx \quad u = x^2 + 1$$

$$u - 1 = x^2$$

$$du = 2x dx$$

$$\frac{1}{2} \int \frac{(u-1)}{u} du$$

$$\frac{1}{2} \int \left(1 - \frac{1}{u}\right) du$$

$$\frac{1}{2} [u - \ln|u|] + c$$

$$\frac{1}{2} (x^2 + 1) - \frac{1}{2} \ln(x^2 + 1) + c$$

$$\textcircled{15} \int x^2 \sqrt{4x^3 + 5} dx \quad u = 4x^3 + 5$$

$$du = 12x^2 dx$$

$$\frac{1}{12} \int u^{1/2} du$$

$$\frac{1}{18} (4x^3 + 5)^{3/2} + c$$

$$\textcircled{16} \int x^2 \sec^2(x^3) dx$$

$$\frac{1}{3} \tan(x^3) + c$$

$$\textcircled{17} \int x^4 \sqrt{x^5 + 6} dx \quad u = x^5 + 6$$

$$du = 5x^4 dx$$

$$\frac{1}{5} \int u^{1/2} du$$

$$\frac{2}{25} (x^5 + 6)^{5/2} + c$$

$$\textcircled{18} \int \frac{\cos x}{\sin^4 x} dx \quad u = \sin x$$

$$du = \cos x dx$$

$$\int u^{-4} du$$

$$-\frac{1}{3} (\sin x)^{-3} + c$$

$$-\frac{1}{3} \csc^3 x + c$$

$$\textcircled{19} \int \frac{(1+\sqrt{x})^3}{\sqrt{x}} dx \quad u = 1 + \sqrt{x}$$

$$du = \frac{1}{2} x^{-1/2} dx$$

$$2 \int u^3 du$$

$$\frac{1}{2} (1 + \sqrt{x})^4 + c$$

$$\textcircled{20} \int x^3 \sqrt{x^2 + 2} dx \quad u = x^2 + 2$$

$$u - 2 = x^2$$

$$du = 2x dx$$

$$\frac{1}{2} \int (u-2) u^{1/2} du$$

$$\frac{1}{2} \int (u^{3/2} - 2u^{1/2}) du$$

$$\frac{1}{5} (x^2 + 2)^{5/2} - \frac{2}{3} (x^2 + 2)^{3/2} + c$$

$$\textcircled{21} \int \frac{x}{\sqrt{1+x^2}} dx \quad u=1+x^2 \quad du=2x dx$$

$$\frac{1}{2} \int u^{-1/2} du$$

$$\sqrt{1+x^2} + C$$

$$\textcircled{22} \int x(1-x^2)^{15} dx$$

$$= \frac{1}{2} \int -2x(1-x^2)^{15} dx$$

$$= -\frac{1}{32} (1-x^2)^{16} + C$$

$$\textcircled{23} \int \frac{2x+3}{(x^2+3x+5)^4} dx$$

$$= -\frac{1}{3} (x^2+3x+5)^{-3} + C$$

$$\textcircled{24} \int (x+2)(x^2+4x+5)^6 dx$$

$$\frac{1}{14} (x^2+4x+5)^7 + C$$

$$\textcircled{25} \int \frac{x+1}{\sqrt[3]{3x^2+6x+5}} dx \quad u=3x^2+6x+5 \quad du=6x+6 dx$$

$$\frac{1}{6} \int u^{-1/3} du$$

$$\frac{1}{4} (3x^2+6x+5)^{2/3} + C$$

$$\textcircled{26} \int \frac{e^{2x}}{(1+e^{2x})^3} dx \quad u=e^{2x}+1 \quad du=2e^{2x} dx$$

$$\frac{1}{2} \int \frac{1}{u^3} du$$

$$= -\frac{1}{4} (e^{2x}+1)^{-2} + C$$

$$\textcircled{27} \int \frac{e^x}{(2e^x+3)^2} dx \quad u=2e^x+3 \quad du=2e^x dx$$

$$\frac{1}{2} \int u^{-2} du$$

$$= -\frac{1}{2} (2e^x+3)^{-1} + C$$

$$\textcircled{28} \int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$$

$$2 \int \frac{1}{2} \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$$

$$2e^{\sqrt{x}} + C$$

$$\textcircled{29} \int \frac{2x+3}{(x+1)^2} dx \quad u=x+1 \quad x=u-1 \quad du=dx$$

$$\int \frac{2u+1}{u^2} du$$

$$\int (2u^{-1} + u^{-2}) du$$

$$2 \ln|x+1| - \frac{1}{x+1} + C$$

$$\textcircled{30} \int \frac{x^2}{x-3} dx \quad u=x-3 \quad x=u+3 \quad du=dx$$

$$\int \frac{(u+3)^2}{u} du$$

$$\int \frac{u^2+6u+9}{u} du$$

$$\int (u+6+9u^{-1}) du$$

$$\frac{1}{2} (x-3)^2 + 6(x-3) + 9 \ln|x-3| + C$$

$$\textcircled{31} \int \tan x dx$$

$$\int \frac{\sin x}{\cos x} dx$$

$$= -\ln|\cos x| + C$$

$$\textcircled{32} \int \sec x \tan x dx$$

$$\sec x + C$$

$$\textcircled{33} \int \sec x \tan x \sqrt{1+\sec x} dx$$

$$\int u^{1/2} du$$

$$\frac{2}{3} (1+\sec x)^{3/2} + C$$

$u=1+\sec x$
 $du=\sec x \tan x dx$

$$(34) \int \frac{5x}{3x^2+4} dx \quad u = 3x^2+4 \\ du = 6x dx$$

$$\frac{5}{6} \int \frac{6x}{3x^2+4} dx$$

$$\frac{5}{6} \ln(3x^2+4) + C$$

$$(35) \int \tan^2 x \csc x dx$$

$$\int \sec x \tan x dx$$

$$\sec x + C$$

$$(36) \int x \tan(x^2) dx$$

$$\frac{1}{2} \int 2x \tan(x^2) dx$$

$$-\frac{1}{2} \ln|\cos(x^2)| + C$$

$$(37) \int \ln(\cos x) \tan x dx$$

$$-\frac{1}{2} [\ln(\cos x)]^2 + C$$

$$(38) \int \frac{dx}{\sqrt{x}(\sqrt{x}+2)^3} \quad u = \sqrt{x}+2 \\ du = \frac{1}{2} x^{-1/2} dx$$

$$2 \int u^{-3} du$$

$$-(\sqrt{x}+2)^{-2} + C$$

$$(39) \int \frac{e^x}{e^{2x}+2e^x+1} dx$$

$$\int \frac{e^x}{(e^x+1)^2} dx \quad u = e^x+1 \\ du = e^x dx$$

$$\int u^{-2} du$$

$$-\frac{1}{e^x+1} + C$$

$$(40) \int \frac{e^{\tan x}}{1-\sin^2 x} dx$$

$$\int \frac{e^{\tan x}}{\cos^2 x} dx$$

$$\int e^{\tan x} \sec^2 x dx$$

$$e^{\tan x} + C$$