# Calculus – Integrals – Lesson 1

I. Find the area beneath the curve for the given interval by the given method.

1. $y = x^2 + 2x$ for $[1, 2]$ with 4 rectangles

2. $y = \dfrac{1}{x+1}$ for $[0, 1]$ with 3 trapezoids

3. $y = x^3 + 1$ for $[0, 1]$ with 3 rectangles

4. $y = \sqrt{x+1}$ for $[0, 3]$ with 6 trapezoids
(use a calculator, round to 3 decimal places)

II. Integrate.

5. $\int (x^2 - 4)^2 \, dx$

6. $\int \left( x^3 - 4x^2 + 6x - 7 + 8x^{-2} - 3x^{\frac{1}{6}} \right) dx$

7. $\int (x^3 - 4x)(x^6 - 5x) \, dx$

8. $\int \dfrac{6x^3 - 4x^2 + 8x - 10}{2x} \, dx$

9. $\int (x+2)^5 \, dx$

10. $\int_0^2 (4x^3 - 6x^2 + 2x - 10) \, dx$

11. $\int_{-2}^{-1} (3x^2 - 4x + 2x^{-1}) \, dx$

12. $\int_1^4 \pi (6x^2 - 2x + 1) \, dx$

III. Concept problems.

13. Suppose that $\int_0^2 f(x)\,dx = 2$; $\int_1^2 f(x)\,dx = -1$; and $\int_2^4 f(x)\,dx = 7$

Evaluate.

a. $\int_1^4 f(x)\,dx$

b. $\int_0^4 3f(x)\,dx$

c. $\int_0^1 f(x)\,dx$

d. $\int_0^1 f(x+1)\,dx$

e. $\int_0^1 \big(f(x)+1\big)\,dx$

f. $\int_2^4 f(x-2)\,dx$

g. $\int_2^4 \big(f(x)-2\big)\,dx$

h. Explain why $f$ must be negative somewhere in the interval [1, 2].

i. Explain why $f(x) \geq 3$ somewhere in the interval [2, 4].

j. Draw the graph of a function $f$ that satisfies the initial requirements.

14. Evaluate each of the following integrals using graphical arguments and the fact that $\int_0^{\pi} \sin x\,dx = 2$.

a. $\int_0^{2\pi} \sin x\,dx$

b. $\int_0^{2\pi} |\sin x|\,dx$

c. $\int_0^{\pi} (1+\sin x)\,dx$

d. $\int_0^{\frac{\pi}{2}} \sin x\,dx$

e. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x\,dx$

# Calculus – Integrals – Lesson 2

**I.** **Examples** – Find the area bounded by the curves.

1. $y = x^3 - 8x + 10, \quad x = 1, \quad x = 2, \quad x-axis$

2. $y = x^2 - 5x - 6, \quad x-axis$

3. $y = 4x - x^3, \quad x-axis$

4. $y = 11 - x^2, \quad y = x^2 - 4x + 5$

**II.** **Problems** – Find the area bounded by the curves.

1. $y = 3x + 2, \quad x = 1, \quad x = 3, \quad x-axis$

2. $y = -x^2 + x + 2, \quad x-axis$

3. $y = x^3 + 5x^2, \quad x-axis$

4. $y = x^4 - 5x^2 + 4, \quad x-axis$

5. $y = x^2, \quad y = x$

6. $y = 8 + 2x - x^2, \quad x - y + 2 = 0$

7. $\sqrt{x} + \sqrt{y} = 4, \quad x-axis, \quad y-axis$

8. $x^2 + y^2 = 25, \quad y-axis, \quad x = 3$

9. $x^2 y - x^3 + 3 = 0, \quad x = 2, \quad x = 10, \quad x-axis$

10. $y = x^2 + 3x + 3, \quad$ *and the chord joining* $(-2, 1)$ *and* $(1, 7)$.

III. Concept problem

11. Evaluate $\int_0^1 \sqrt{1-(x-1)^2}\,dx$ (Hint: Sketch a graph of the integrand.)

# Calculus – Integrals – Lesson 3

Integration of $\int f(x)^n \, f'(x) =$

I. Examples

1. $\int (3x+1)^4 \, dx$
2. $\int 9(x^2-5)^{\frac{1}{2}} \, dx$
3. $\int \sqrt{x}\left(x^{\frac{3}{2}}+7\right)^6 \, dx$
4. $\int (x^2+2x)(x^3+3x^2-1)^3 \, dx$
5. $\int (x^6-1)^5 \, dx$
6. $\int x^7(x^2+2)^8 \, dx$

II. Integrate

1. $\int \sqrt{x-2} \, dx$
2. $\int (2x+3)^{12} \, dx$
3. $\int \sqrt{3t+2} \, dt$
4. $\int (2x+3)^{43} \, dx$
5. $\int 3(4x-5)^{\frac{2}{3}} \, dx$
6. $\int 2(3-5x)^{\frac{3}{5}} \, dx$
7. $\int \sqrt[3]{4x+7} \, dx$
8. $\int \frac{dx}{(2x-3)^4}$
9. $\int y(y^2+4)^{\frac{2}{3}} \, dx$
10. $\int 5x(1-x^2)^{\frac{1}{2}} \, dx$
11. $\int x\sqrt{2x^2+7} \, dx$
12. $\int x^2\sqrt{2x^3-7} \, dx$
13. $\int 5x^2(3-2x^3)^{\frac{1}{3}} \, dx$
14. $\int x^3\sqrt{x^4-16} \, dx$
15. $\int \frac{t \, dt}{\sqrt{3t^2+1}}$
16. $\int x^{\frac{1}{3}}\left(x^{\frac{4}{3}}+81\right)^6 \, dx$
17. $\int \sqrt{1+\sqrt{x}} \, \frac{dx}{\sqrt{x}}$
18. $\int x^{-\frac{1}{2}}\left(x^{\frac{1}{2}}+2\right)^{\frac{1}{5}} \, dx$
19. $\int x^6(3x^2+1)^{15} \, dx$
20. $\int (2x+1)\sqrt{x^2+x-3} \, dx$
21. $\int (x+5)\sqrt{x^2+10x+24} \, dx$
22. $\int (x^6-7x^4+2)^{20} \, dx$
23. $\int \left(\sqrt[3]{x^2}+3\right)^5 \, \frac{dx}{\sqrt[3]{x}}$
24. $\int \left((x+1)^5-2\right)^6(x+1)^4 \, dx$
25. $\int \sqrt{x}\left(\sqrt{x}+2\right)^{199} \, dx$
26. $\int \sqrt[4]{\left(\sqrt{x}-1\right)^3+2} \quad \left(\sqrt{x}-1\right)^2 \, \frac{dx}{\sqrt{x}}$
27. $\int \frac{x-5}{\sqrt{x+2}} \, dx$
28. $\int (x+1)(x-2) \, dx$
29. $\int \frac{5x^3-10x^2+15}{5x^6} \, dx$
30. $\int \frac{dx}{(x-5)^3}$

# Calculus – Integrals – Lesson 4

Integrate.

1. $\int \cos(2x+3)\,dx$
2. $\int \sin(2-3x)\,dx$
3. $\int (3x-2)^4\,dx$
4. $\int x\cos(1-x^2)\,dx$
5. $\int \frac{2x^3}{1+x^4}\,dx$
6. $\int x(3x+2)^4\,dx$
7. $\int \frac{dx}{1-2x}$
8. $\int \sqrt{3x-2}\,dx$
9. $\int x\sqrt{3-2x}\,dx$
10. $\int \frac{\ln x}{x}\,dx$
11. $\int \frac{\sqrt{1+\frac{1}{x}}}{x^2}\,dx$
12. $\int x\,e^{x^2}\,dx$
13. $\int x^3(x^4-1)^2\,dx$
14. $\int \frac{x^3}{1+x^2}\,dx$
15. $\int x^2\sqrt{4x^3+5}\,dx$
16. $\int x^2\sec^2(x^3)\,dx$
17. $\int x^4\sqrt[3]{x^5+6}\,dx$
18. $\int \frac{\cos x}{\sin^4 x}\,dx$
19. $\int \frac{(1+\sqrt{x})^3}{\sqrt{x}}\,dx$
20. $\int x^3\sqrt{x^2+2}\,dx$
21. $\int \frac{x}{\sqrt{1+x^2}}\,dx$
22. $\int x(1-x^2)^{15}\,dx$
23. $\int \frac{2x+3}{(x^2+3x+5)^4}\,dx$
24. $\int (x+2)(x^2+4x+5)^6\,dx$
25. $\int \frac{x+1}{\sqrt[3]{3x^2+6x+5}}\,dx$
26. $\int \frac{e^{2x}}{(1+e^{2x})^3}\,dx$
27. $\int \frac{e^x}{(2e^x+3)^2}\,dx$
28. $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$
29. $\int \frac{2x+3}{(x+1)^2}\,dx$
30. $\int \frac{x^2}{x-3}\,dx$
31. $\int \tan x\,dx$
32. $\int \sec x\tan x\,dx$
33. $\int \sec x\tan x\sqrt{1+\sec x}\,dx$
34. $\int \frac{5x}{3x^2+4}\,dx$
35. $\int \tan^2 x\csc x\,dx$
36. $\int x\tan(x^2)\,dx$
37. $\int \ln(\cos x)\tan x\,dx$
38. $\int \frac{dx}{\sqrt{x}(\sqrt{x}+2)^3}$
39. $\int \frac{e^x}{e^{2x}+2e^x+1}\,dx$
40. $\int \frac{e^{\tan x}}{1-\sin^2 x}\,dx$

# Calculus – Integrals – Lesson 5

## I. Trig-Substitutions

1. If you have a factor of $a^2 - b^2x^2$ try $x = \frac{a}{b}\sin\theta$.
2. If you have a factor of $a^2 + b^2x^2$ try $x = \frac{a}{b}\tan\theta$.
3. If you have a factor of $b^2x^2 - a^2$ try $x = \frac{a}{b}\sec\theta$.

## II. Examples

1. $\int \frac{dx}{\sqrt{1-9x^2}}$
2. $\int \frac{dx}{x^2+25}$
3. $\int \frac{dx}{x^2\sqrt{x^2-1}}$

## III. Problems

1. $\int \frac{dx}{x^2\sqrt{4-x^2}}$
2. $\int \frac{x^2}{x^2+9}\,dx$
3. $\int \frac{\sqrt{4x^2-25}}{x}\,dx$
4. $\int \sqrt{100-x^2}\,dx$
5. $\int \frac{dx}{x^2\sqrt{4x^2-9}}$
6. $\int \frac{dx}{\left(4+x^2\right)^2}$
7. $\int \frac{x^2}{\sqrt{9-x^2}}\,dx$
8. $\int \frac{dx}{\left(3+x^2\right)^{\frac{3}{2}}}$
9. $\int \frac{x^3}{\sqrt{x^2-25}}\,dx$
10. $\int (2x-5)^{10}\,dx$
11. $\int \sin(4x)\,dx$
12. $\int \frac{dx}{\sin^2 x}$
13. $\int \cos^3 x \sin x\,dx$
14. $\int x^2\left(x^3-5x\right)dx$
15. $\int \frac{x+3}{\sqrt{x-5}}\,dx$
16. $\int 14x^7\left(x^4+2\right)^6\,dx$
17. $\int (1+\tan x)^{\frac{1}{3}} \sec^2 x\,dx$
18. $\int \sec^2\sqrt{x}\,\frac{dx}{\sqrt{x}}$
19. $\int \frac{x^3-4x}{\sqrt{x}}\,dx$
20. $\int x\sec^4\left(9x^2\right)dx$

# Calculus – Integral Review

I. Find the area beneath the curve by the given method.

1. Find the area beneath $y = \frac{1}{x}$ over the interval $[1,3]$ using 4 rectangles.
2. Find the area beneath $y = x^2 + 2$ over the interval $[0,2]$ using 6 trapezoids.

II. Find the area bounded by the given curves.

3. $y = x^3 - 2x + 1,\ y = -2x,\ x = 1$
4. $y = \frac{1}{1+x^2},\ y = \frac{x^2}{2}$
5. $y = x(x+1)^{\frac{1}{3}},\ x - axis$

III. Integrate by the most appropriate method.

6. $\int x^2\sqrt{x^3-1}\,dx$
7. $\int \frac{1}{\sqrt{x}\left(\sqrt{x}+1\right)^3}\,dx$
8. $\int \sec^2 5x\,dx$
9. $\int \left(8x^4 - 3x^{-2} + x^{\frac{1}{2}}\right)dx$
10. $\int x^2(x+1)(x-2)\,dx$
11. $\int \frac{5x^2}{x^3+4}\,dx$
12. $\int \cot^7 x \csc^2 x\,dx$
13. $\int \left(1+\sqrt{\cos x}\right)^2 \sin x\,dx$
14. $\int \left(x^4+3\right)^9 x^7\,dx$
15. $\int \sqrt{x^5-4}\,x^3\,dx$
16. $\int \frac{x^2}{\sqrt{49-x^2}}\,dx$
17. $\int \frac{x-2}{\left(x^2-4x+3\right)^3}\,dx$
18. $\int \tan x^3 \sec x^3\ x^2\,dx$
19. $\int \cos^2(8x+2)\,dx$
20. $\int \frac{x^2-4x}{\sqrt[3]{x^2}}\,dx$
21. $\int e^{9x^2} x\,dx$
22. $\int \tan^3 x \sec^4 x\,dx$
23. $\int \frac{1+\sin x\cos x}{\left(2x+\sin^2 x\right)^4}\,dx$
24. $\int \frac{x\,dx}{\sqrt{x^2-100}}$
25. $\int \frac{\left(e^{3x}+1\right)^2}{e^x}\,dx$