

HW 54 – Cumulative Review #1  
AB Calculus – Mr. Oberle

Write a complete, detailed solution to each problem.

1. [No calculator] Let  $f$  be a function defined by  $f(x) = \begin{cases} 2x+1 & x \leq 2 \\ \frac{1}{2}x^2 + k & x > 2 \end{cases}$ .

- For what value of  $k$  will  $f$  be continuous at  $x = 2$ ? Justify your answer.
- Using the value of  $k$  found in part (a), determine whether  $f$  is differentiable at  $x = 2$ . Use the definition of the derivative to justify your answer.
- Let  $k = 4$ . Determine whether  $f$  is differentiable at  $x = 2$ . Justify your answer.

2. [No calculator] Let  $h$  be a function defined for  $x \neq 0$  such that  $h(4) = -3$  and the derivative of  $h$  is given by

$$h'(x) = \frac{x^2 - 2}{x} \text{ for all } x \neq 0.$$

- Find all values of  $x$  for which the graph of  $h$  has a horizontal tangent, and determine whether  $h$  has a local maximum, a local minimum, or neither at each of these values. Justify your answers.
- On what intervals, if any, is the graph of  $h$  concave up? Justify your answer.
- Write an equation for the line tangent to the graph of  $h$  at  $x = 4$ .
- Does the line tangent to the graph of  $h$  at  $x = 4$  lie above or below the graph of  $h$  for  $x > 4$ ? Why?

3. The rate at which water flows out of a pipe, in gallons per hour, is given by a differentiable function  $R$  of time  $t$ . The table at right shows the rate as measured every 3 hours for a 24-hour period.

- Use a midpoint Riemann sum with 4 subdivisions of equal length to approximate  $\int_0^{24} R(t)dt$ . Using correct units, explain the meaning of your answer in terms of water flow.
- Is there some time  $t$ ,  $0 < t < 24$ , such that  $R'(t) = 0$ ? Justify your answer.
- The rate of water flow  $R(t)$  can be approximated by

$$Q(t) = \frac{1}{79}(768 + 23t - t^2).$$

Use  $Q(t)$  to approximate the average rate of water flow during the 24-hour time period. Indicate units of measure.

$t$ (hours)	$R(t)$ (gallons per hour)
0	9.6
3	10.4
6	10.8
9	11.2
12	11.4
15	11.3
18	10.7
21	10.2
24	9.6