HW 1 – Parametric Equations; The Cycloid Family of Curves Name:
Multivariable Analysis – Mr. Oberle

Graph the curve either by hand by plotting points or using your calculator on the indicated interval. You must indicate the orientation of the curve on your graph.

1. [by hand] $x = t^2 - 4,\ y = t^3 - 4t$ for $-3 \le t \le 3$
2. [calculator] $x = 2\sin 4t,\ y = 3\cos 3t$ for $0 \le t \le 2\pi$.
3. [calculator] $x = t + \sin 2t,\ y = t + \sin 3t$ for $0 \le t \le 2\pi$.
4. [calculator] $x = t^3 - 2t,\ y = t^2 - t$ for $-2 \le t \le 2$.

Graph the curve either by hand by plotting points or using your calculator on the indicated interval. You must indicate the orientation of the curve on your graph. Afterwards, find the rectangular equation (i.e. in terms of only x and y) for this graph.

5. [by hand] $x = 2t - 1,\ y = 4 - 3t$ for $-1 \le t \le 3$
6. [calculator] $x = 3\sin t,\ y = 5\cos t$ for $0 \le t \le 2\pi$.
7. [calculator] $x = \tan t,\ y = \sec t$ for $0 \le t \le 2\pi$.

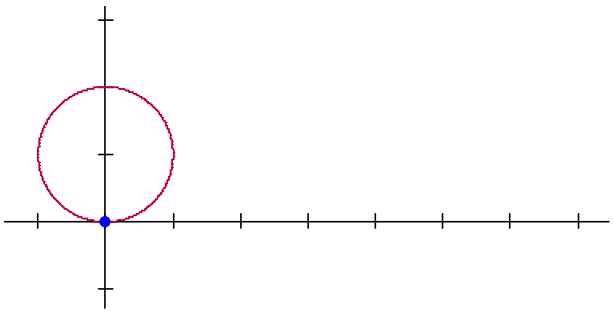
Situation

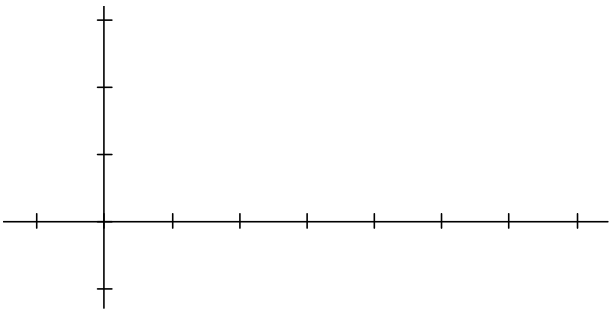
A disk rolls along a flat surface without slipping. A fixed point P on the disk traces out a curve is called a ***cycloid***.

Problem

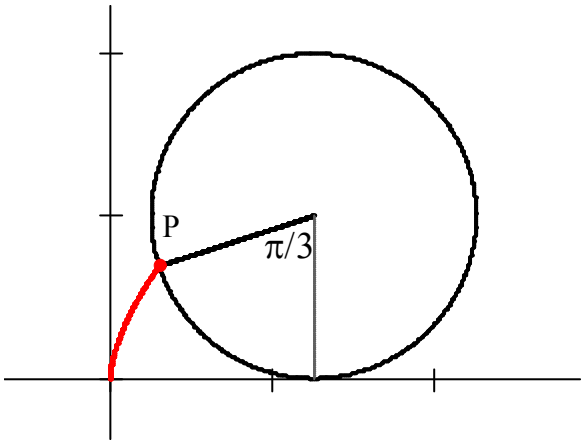
Find the parametric equations for a standard cycloid rolling along the x-axis. The disk has radius 1, its center is initially (0, 1), and the point P starts out at the origin.

8. Think about the situation described above. What do you think the path traced by the point will look like? Make a sketch of path you think the point will take.

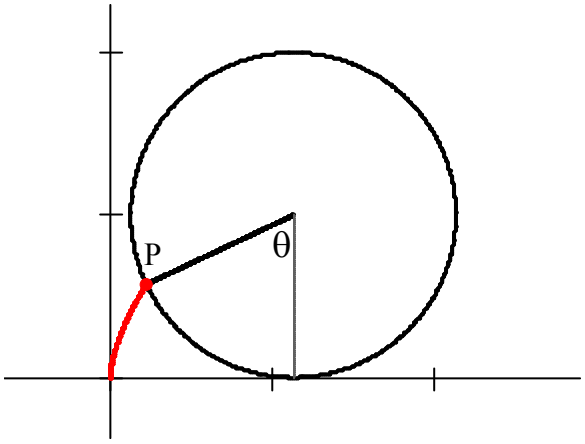




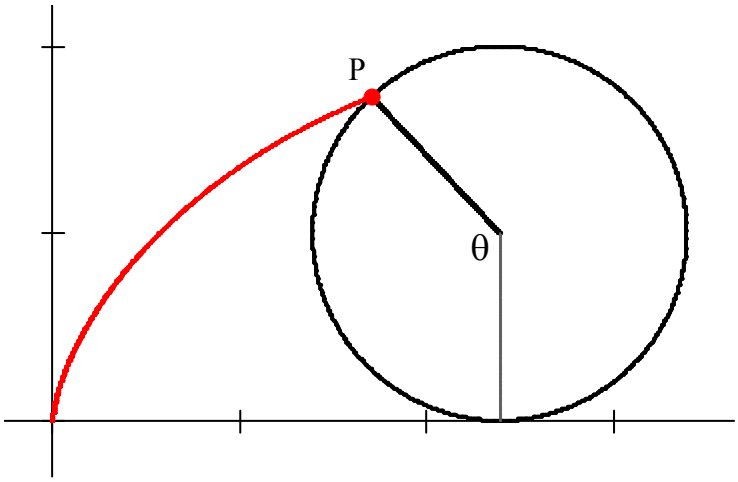
9. Find the coordinates of the **center** of the circle below. Then use that point to find the coordinates of the point P.



10. Find the coordinates of point P below in terms of θ.



11. You now have the parametric equations, in terms of θ, for a cycloid given that $0 \leq \theta \leq \pi/2$. Will this set of equations work for other values of θ as well? Verify that your equations from the previous problems also work for $\pi/2 \leq \theta \leq \pi$ (see the picture below).



12. Now find the parametric equations of the cycloid constructed using a disk of radius a instead of radius 1.

Exploration

13. Suppose that point P, instead of being on the disk's circumference, is on the radius at distance d (where $0 < d < a$) from the center. Find the parametric equations for the path taken by P as the disk rolls along the x-axis. This figure is called a **trochoid**.

14. Suppose that, in the previous problem, P is on the radius at distance d from the center but that $d > a$. Now what are the parametric equations for the path taken by P? As in the previous problem, this curve is also called a **trochoid**.

Getting to know technology

15. Draw the graph of at least one cycle of a cycloid on your TI graphing calculator. Make sure it looks nice, and copy your graph below.

16. Draw at least one arc of a cycloid using Winplot. Email a copy of your document to me.

This handout was taken almost in its entirety from a handout by Bruce Simmons.